Convection in Eccentric Annuli with Inner Cylinder Rotation

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Abstract

A NUMERICAL model was developed to study the effects of rotation on the fluid motion and heat-transfer processes in the annular space between eccentric cylinders when the inner cylinder is heated and rotating. Experimental verification is based on the calorimetric technique. Comparison is made between the numerical and experimental results for a moderate radius ratio of 2.6 with eccentricities of 0, 1/3, and 2/3. The overall equivalent thermal conductivity (K_{eq}) is obtained for Rayleigh numbers Ra up to 10^6 with rotational Reynolds number Re variations of 0-1120. The Prandtl number Pr of the fluid chosen is 0.7 for air. Investigation shows that, for Re up to the order of 10^2, the numerical model shows promising results when Ra is increased.

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A schematic of the annular configuration investigated here is shown in Fig. 1a. Fluid motion in the annular space is assumed to be two-dimensional and laminar. It is assumed that the end effects are negligible and that the Boussinesq approximation is valid. With these assumptions, the dimensionless governing equations for vorticity ζ, stream function ψ, and temperature θ are

$$(\partial\zeta/\partial t) + \nabla\cdot(\zeta \underline{U}) = -Pr\,Ra\,\nabla\times(\theta\hat{g}) + Pr\nabla^2\zeta \qquad (1)$$

$$\nabla^2\psi = -\zeta \qquad (2)$$

$$(\partial\theta/\partial t) + \nabla\cdot(\theta\underline{U}) = \nabla^2\theta \qquad (3)$$

where t and $\hat{g}$ are the time and gravity vector, respectively. Rayleigh number $Ra = g\beta\Delta TL^3/\alpha\nu$, Reynolds number $Re = r_i\Omega_i L/\nu$, and Prandtl number $Pr = \nu/\alpha$. β is the coefficient of volumetric expansion and ΔT the temperature difference between inner and outer cylinders. $L = (r_o - r_i)$, where r_i and r_o are the radius of the inner and outer cylinders, respectively. Ω_i is the angular rotational rate of the inner cylinder and α and ν the thermal diffusivity and kinematic viscosity, respectively.

Velocities in dimensionless form are given by $U_x = \partial\psi/\partial y$ and $U_y = -\partial\psi/\partial y$. Transformation of the Cartesian $x-y$ coordinate system to a bipolar $\zeta-\eta$ coordinate system is given by

$$x = c\,\sinh\eta/(\cosh\eta - \cos\zeta) \qquad (-\infty < \eta < \infty)$$

$$y = c\,\sin\zeta/(\cosh\eta - \cos\zeta) \qquad (0 \le \zeta \le 2\pi) \qquad (4)$$

where c is a scaling factor of the transformation related to the eccentricity ratio e and the radius ratio of the two cylinders.

A schematic of the experimental test setup is shown in Fig. 1(b) and described in detail by Yeo.[1] The experimental results and the predictions of the numerical model were initially tested with the cylinders stationary. Results on K_{eq} vs Ra for stationary concentric cylinders of radius ratio 2.6 correlated well with those of Kuehn and Goldstein.[2,3] When the inner cylinder is rotated, say counterclockwise, Fig. 2 shows that the immediate effect is to cause the hot fluid adjacent to the surface of the inner cylinder to move with it by virtue of viscous drag. The hot fluid on the ascending side of the cylinder, therefore, rises even faster while, on the descending side, the rising hot fluid is slowed. Near the surface of the inner cylinder on the descending side, there is a small region with very low flow velocity. The rotating cell on the ascending side of the inner cylinder is also progressively elongated, while the cell on the descending side is compressed. The effects of these flow patterns on the heat-transfer characteristic of the cylinders are shown in Fig. 3a for $e = 0$. Experimental results for the overall heat-transfer coefficient are also shown. The agreement is good at low rotational Reynolds numbers. At the Rayleigh number of 2.6×10^4, deviation from the numerical prediction starts around a Reynolds number of about 120. As the Rayleigh number increases to 5.5×10^4, the results agree up to a Reynolds number of about 180. A qualitative comparison of the trends indicates that the deviation at a Rayleigh number of 9.9×10^4 occurs at a higher Reynolds number of around 250. Anderson and Saunders[4] theorized that deviation should set in when the surface velocity of the heated cylinder is approximately equal to the upward free convection velocity at the side of the cylinder when the cylinder is stationary. This means that the flow around the heated cylinder should become unstable when the "root" of the thermal plume is approximately at the level of the center of the cylinder.

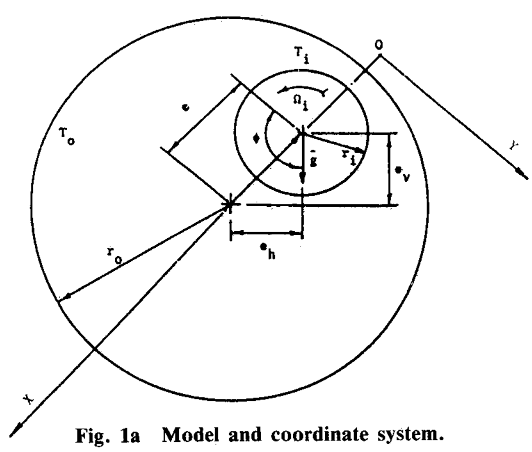

Fig. 1a Model and coordinate system.

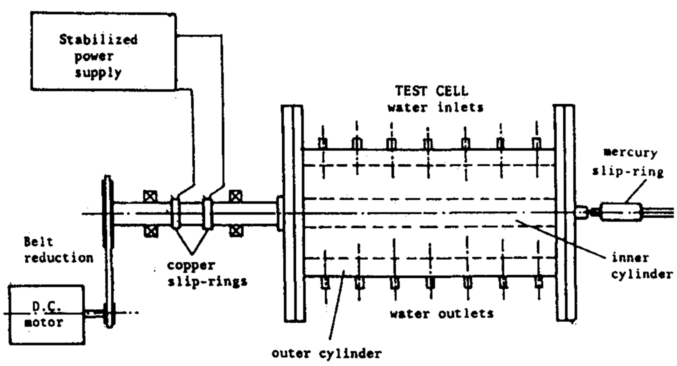

Fig. 1b Schematic of experimental setup.

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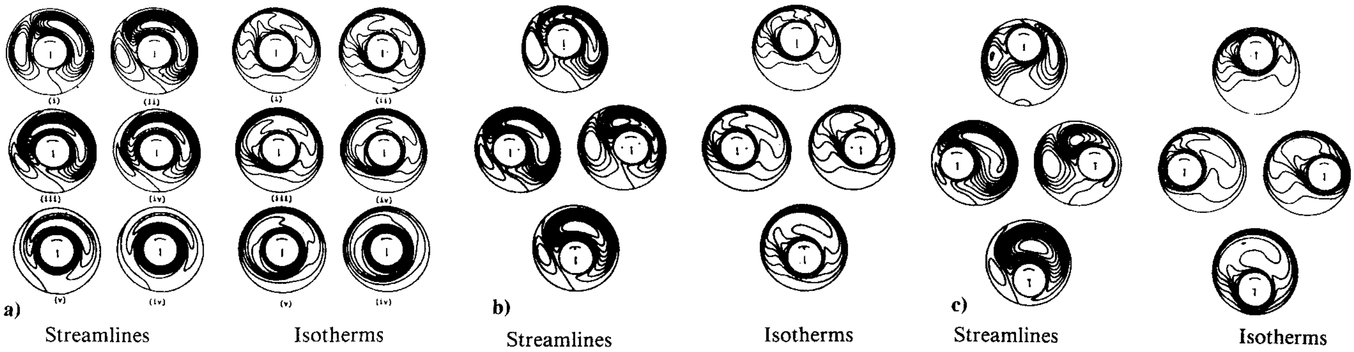

Fig. 2 Streamlines and isotherms, $r_0/r_i = 2.6$, $Ra = 5.5 \times 10^4$. a) $e = 0$; $Re = 140, 280, 420, 560, 800, 1120$ for (i), (ii), (iii), (iv), (v) and (vi), respectively. b) $e = 1/3$; $Re = 280$ at $\phi = 0, 90, 180,$ and 270 deg. c) $e = 2/3$; $Re = 280$ at $\phi = 0, 90, 180,$ and 270 deg.

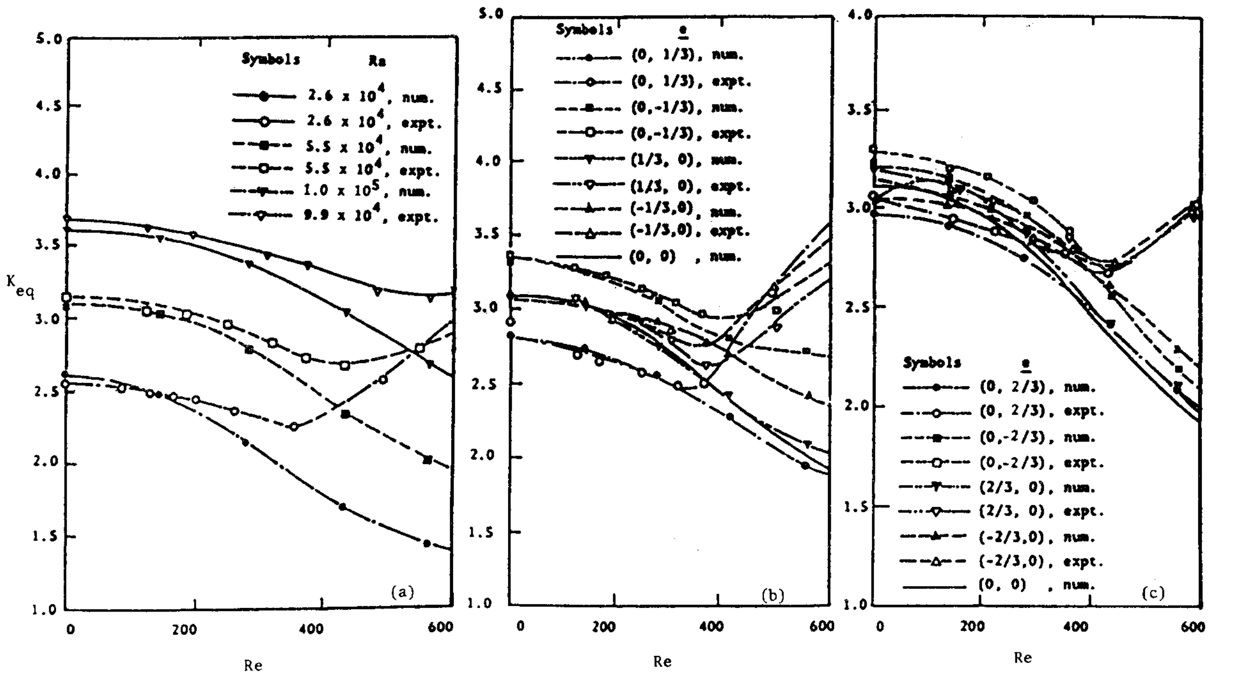

Fig. 3 Effects of rotation on the overall equivalent thermal conductivity K_{eq} at various Ra for $r_0/r_i = 2.6$ at a) $e = 0$, b) $e = 1/3$, and c) $e = 2/3$.

Experimental studies at eccentric positions for eccentricity ratios of 1/3 and 2/3 (Figs. 3b and 3c) show essentially the same trends of behavior as that for the concentric position, in that the overall heat-transfer coefficient K_{eq} falls with increasing Reynolds number to a minimum point. The points of minimum K_{eq} at the eccentric positions studied ($e = 1/3$ and 2/3) occur around the same Reynolds number as that for the concentric case at the same Rayleigh number, except that beyond the minimum point the experimental K_{eq} rises at a faster rate when the eccentricity ratio is higher.

There are striking similarities between the interference photographs of Etemad[5] and the numerical isotherms shown in Fig. 2, illustrating the effects of the increasing Reynolds number at a fixed Rayleigh number and different eccentricities. The Etemad's photographs show the shifting of the thermal plume and its "root" in the direction of rotation when the inner cylinder is rotated. When the "root" of the thermal plume is moved below the level of the center of the inner cylinder, it becomes diffused and the plume itself becomes ill-defined. In the vicinity of the inner heated rotating cylinder, there is a rapid thickening of the thermal boundary layer and the formation of distinct rings of widely spaced isotherms that are not unlike those found in this investigation. This suggests that the present laminar model yields a good approximation of the two-dimensional pictures of the real flow at low Re and high Ra.

References

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